3.4 homework

5, 9, 8

5) $f(x) = (x^3)(\sin x) - 5(\cos x)$

$f'(x) = (3x^2)(\sin x) + (x^3)(\cos x) - 5(-\sin x)$

$f'(x) = 3x^2 \sin x + x^3 \cos x + 5 \sin x$

$= \left[3x^2 \sin x + 5 \sin x\right] + x^3 \cos x$

$\sin x \left[\frac{3x^2 \sin x}{\sin x} + \frac{5 \sin x}{\sin x}\right] + x^3 \cos x$

$= (\sin x)\left[3x^2 + 5\right] + x^3 \cos x$

$\frac{d}{dx}(5 \cos x)$

$\underbrace{\frac{d}{dx}(5)}_{0}(\cos x) + (5)\frac{d}{dx}(\cos x)$

$5(-\sin x)$

$\frac{d}{dx}(c f(x))$

$= c \frac{d}{dx}(f(x))$

8) $(x^2+1)(\sec x)$

$y' = (2x)(\sec x) + (x^2+1)(\sec x)(\tan x)$

9) $f(x) = (\sec x)(\tan x)$

$f'(x) = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$

$= \sec x \tan^2 x + \sec^3 x$

27) $y = x\sin x$ is a solution to $y'' + y = 2\cos x$

$y = (x)(\sin x)$

$y' = (1)\sin x + x(\cos x)$

$= \sin x + x\cos x$

$y'' = \cos x + [(1)\cos x + x(-\sin x)]$

$= 2\cos x - x\sin x$

$y'' + y = (2\cos x - x\sin x) + (x\sin x) = 2\cos x$ ✓

b) show $y^{(4)} + y'' = -2\cos x$

$y'' = 2\cos x - x\sin x$

$y''' = -2\sin x - [\sin x + x\cos x]$

$= -3\sin x - x\cos x$

$y'''' = y^{(4)} = -3\cos x - [\cos x - x\sin x]$

$= -4\cos x + x\sin x$

$y^{(4)} + y'' = (-4\cos x + x\sin x) + (2\cos x - x\sin x)$

$= -2\cos x$ ✓

3.4 homework

2010-10-06 Pd 2

13) find deriv. of $\dfrac{\cot x}{1+\csc x}$

$$y' = \frac{(-\csc^2 x)(1+\csc x) - (\cot x)(-\csc x \cot x)}{(1+\csc x)^2}$$

25) Find the eqn of the line tangent to the graph $y = \tan x$ @

ⓐ $x = 0$

To write eqn of a line, I NEED:

slope — think "value of derivative"

point @ $x = 0$

$Pt = (0, \tan 0) = (0, 0)$

$y = \tan x$

$y' = \sec^2 x$

$y'\big|_{x=0} = \sec^2(0) = \frac{1}{\cos^2(0)} = \frac{1}{[\cos(0)]^2} = \left[\frac{1}{\cos(0)}\right]^2$

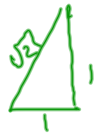
$= \frac{1}{1} = 1$

EQN:

$y - 0 = 1(x - 0)$

or $y = x$

b) @ $x = \frac{\pi}{4}$



slope

$y' = \sec^2 x$

$y'\big|_{x=\frac{\pi}{4}} = \sec^2\left(\frac{\pi}{4}\right)$

$= \frac{1}{\left(\cos\frac{\pi}{4}\right)^2} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{1}{\frac{1}{2}} = 2$

Pt. $\left(\frac{\pi}{4}, \tan\left(\frac{\pi}{4}\right)\right)$

$= \left(\frac{\pi}{4}, 1\right)$

EQN: $y - 1 = 2\left(x - \frac{\pi}{4}\right)$

c) $y = \tan x$; $x = -\frac{\pi}{4}$

$m = y'\big|_{x=-\frac{\pi}{4}} = \sec^2\left(-\frac{\pi}{4}\right)$

$= 2$

$Pt = \left(-\frac{\pi}{4}, \tan\left(-\frac{\pi}{4}\right)\right)$

$= \left(-\frac{\pi}{4}, -1\right)$

EQN: $y - (-1) = 2\left(x - \left(-\frac{\pi}{4}\right)\right)$

26) Find the eqn of tangent line to graph of $y = \sin x$ @

a) $x = 0$

slope: $y' = \cos x$

$m = \cos 0 = 1$

Pt: $(0, 0)$

Eqn:

$y - 0 = 1(x - 0)$

$y = x$

b) $x = \pi$

$m = \cos(\pi) = -1$

$(\pi, 0)$

$y - 0 = -1(x - \pi)$

$y = \pi - x$

c) $x = \frac{\pi}{4}$

$m = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

3.4 homework

2010-10-07 Pd 2

2-10
6, 2, 10

2) $f(x) = (\underset{f}{\sin x})(\underset{g}{\cos x})$

$f'g + fg'$

$\frac{d}{dx}(\sin x)(\cos x) + (\sin x)\frac{d}{dx}(\cos x)$

$(\cos x)(\cos x) + (\sin x)(-\sin x)$ $(\sin x)(\sin x)(-1)$

$= \cos^2 x - \sin^2 x$

(6) $y = \dfrac{\cos x}{x \sin x} = \boxed{\left(\dfrac{1}{x}\right)(\cot x)}$

$$y' = \frac{\frac{d}{dx}(\cos x)(x \sin x) - (\cos x)\frac{d}{dx}(x \sin x)}{(x \sin x)^2}$$

$$= \frac{(-\sin x)(x \sin x) - (\cos x)\big((1)(\sin x) + (x)(\cos x)\big)}{(x \sin x)^2}$$

$$= \frac{-x \sin^2 x - (\sin x)(\cos x) - x \cos^2 x}{(x \sin x)^2}$$

ALT

$$\left(\frac{1}{x}\right)(\cot x) = (x^{-1})(\cot x)$$

$$y' = (-x^{-2})(\cot x) + (x^{-1})(-\csc^2 x)$$

10) $f(x) = \frac{\sec x}{1+\tan x}$

$$f'(x) = \frac{(\sec x \tan x)(1+\tan x) - (\sec x)(\sec^2 x)}{(1+\tan x)^2}$$

25
27

25) $y = \tan x$. Find eqn of tangent line

a) @ $x = 0$

To write eqn of a line you need:

☆ slope = value of the derivative @ $x = 0$

☆ a pt.

$Pt = (0, f(0)) = (0, \tan 0)$
$= (0, 0)$

$y = \tan x$

$y' = \sec^2 x$

$y'\Big|_{x=0} = \sec^2(0) = \dfrac{1}{\cos^2 0} = 1$

$y - 0 = 1(x - 0)$

$y = x$

b) @ $x = \frac{\pi}{4}$

slope $= \sec^2\left(\frac{\pi}{4}\right) = \dfrac{1}{\cos^2\left(\frac{\pi}{4}\right)} = \dfrac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = 2$

$\dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

$Pt: \left(\frac{\pi}{4}, \tan\frac{\pi}{4}\right)$
$= \left(\frac{\pi}{4}, 1\right)$

EQN: $y - 1 = 2\left(x - \frac{\pi}{4}\right)$

c) @ $x = -\frac{\pi}{4}$

$Pt = \left(-\frac{\pi}{4}, \tan -\frac{\pi}{4}\right)$
$= \left(-\frac{\pi}{4}, -1\right)$

$m = \dfrac{1}{\cos^2\left(-\frac{\pi}{4}\right)} = 2$

EQN: $y - (-1) = 2\left(x - \left(-\frac{\pi}{4}\right)\right)$

27) show that $y = x\sin x$ is a solution of $y'' + y = 2\cos x$

$$y = (x)(\sin x)$$

$$y' = (1)\sin x + (x)\cos x = \sin x + x\cos x$$

$$y'' = \cos x + [(1)(\cos x) + (x)(-\sin x)]$$

$$y'' = 2\cos x - x\sin x$$

$$\overbrace{(2\cos x - x\sin x)}^{y''} + \overbrace{(x\sin x)}^{y} = 2\cos x$$

b) show that $y = x\sin x$ is a sol^n of $y^{(4)} + y'' = -2\cos x$

$$y'' = 2\cos x - (x)(\sin x)$$

$$y''' = 2(-\sin x) - [\sin x + x\cos x]$$

$$= -3\sin x - (x)(\cos x)$$

$$y^{(4)} = -3(\cos x) - [\cos x - x\sin x]$$

$$= -4\cos x + x\sin x$$

$$y^{(4)} + y'' = (-4\cos x + x\sin x) + (2\cos x - x\sin x)$$

$$= -2\cos x \ \checkmark$$

3.4 homework

2010-10-07 Pd 3